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ESTIMATION OF EXPECTED TIME TO SEROCONVERSION OF HIV INFECTED USING TWO PARAMETER BURR TYPE X DISTRIBUTION

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Abstract

The increase in the number of persons who are infected by HIV is a matter of concern for the society as well as for the government. HIV infection results in a number of diseases upon the infected. The HIV infection becomes more dangerous due to successive exposure like sexual contacts and sharing of needles which are HIV infected and blood transmission which is contaminated. So, several researchers found out many mathematical models regarding the antigenic diversity which is a vital factor in HIV infection. One such model to be estimate the expected time to seroconversion due to increasing antigenic diversity. A stochastic model is developed in this paper to estimate the expected time to seroconversion under the assumption that the antigenic diversity threshold is a random variable which follows Two Parameter Burr Type X Distribution. The shock model and cumulative damage processes approach is used for this purpose.

Key words: Antigenic diversity, Threshold, Cumulative damage, Expected time and Seroconvertion.

1. Introduction

HIV infection and its progression to AIDS is a matter of great concern because it affects not only the health of an individual but also the length of life. So much of importance is given by medical experts to study the infection from different aspects. One of the important aspects of study it's to find the rate of progression of this infection to what is called the seropositive status. The intensity of progression of this infection depends upon the extent of exposure to this virus. The antigens which create this infection and help to progression are getting accumulated due to successive contacts. One of the important aspects of the study of this infection and its progression is to estimate expected time to seroconversion. In this paper, a stochastic model is developed to determine the expected time to seroconversion under the assumption that the antigenic diversity threshold is a random variable and its follows two parameter Burr type X distribution. May and Anderson (1987) studied about the transmission dynamics of HIV infection. Mode *et al.* (1988) developed a stochastic model of AIDS epidemic. Stilianaks *et al.* (1994) have discussed about the antigenic diversity threshold. Kannan and Sathiyamoorthi (1998) have discussed about the expected time to seroconversion. The concept of shock model and cumulative damage processes by Easary *et al.* (1973) is used in the present study to find the expected time to seroconversion.

2. Assumptions

- There is contribution to the level of antigenic diversity of an infected individual at every epoch of exposure or contact.
- There is a particular level of antigenic diversity is called they antigenic diversity threshold and if the cumulative contribution to antigenic diversity crosses the threshold level the seroconversion takes place.

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- The antigenic diversity threshold is a random variable which follows two parameter Burr type X distribution.
- The inter contact times are random variables which are identically independently distributed.

3. Notation

X_i = A random variable which denotes the contribution antigenic diversity in the i^{th} contact and $X_i \sim g(.)$.

Y = the random variable which denotes the antigenic diversity threshold which has the pdf h (.) and cdf H (.) and $\overline{H}(.) = 1 - H(.)$.

U = is a random variable which denotes the inter arrival times between successive contacts and the pdf is f (.) and cdf F (.) U_i's are identically independently distributed random variables.

4. Results

The Two Parameter Burr – Type X Distribution. The following is the cumulative distribution function (CDF)

$F(x:\alpha,\lambda) = (1 - e^{-(\lambda x)^2})^{\alpha}$; $x > 0,\ \alpha > 0,\ \lambda > 0$.

where α and λ are shape and scale parameters respectively.

Then the Probability density function (PDF) is

$h(x:\alpha,\lambda) = 2\alpha\lambda^2 x\, e^{-(\lambda x)^2}(1 - e^{-(\lambda x)^2})^{\alpha-1}$; $x > 0,\ \alpha > 0,\ \lambda > 0$.

and

$H(x) = (1 - e^{-(\lambda x)^2})^{\alpha}$

$\overline{H}(x) = 1 - H(x)$

$= 1 - (1 - e^{-(\lambda x)^2})^{\alpha}$

Taking the shape parameter as $\alpha = 1$

Put $\alpha = 1$

$= 1 - (1 - e^{-(\lambda x)^2})$

$= 1 - 1 + e^{-(\lambda x)^2}$

$\overline{H}(x) = e^{-(\lambda x)^2}$

The probability that the cumulative antigenic diversity is below the threshold level Y is

$P(\sum_{n=k}^{n} x_i < y) = \int_0^{\infty} g_k(x)\overline{H}(x)dx$

$= \int_0^{\infty} g_k(x)\ e^{-(\lambda x)^2} dx$

$P(\sum_{n=k}^{n} x_i < y) = [\, g^*(\lambda^2)\ ]^k$

Hence $S(t) = P(T > t)$

V (t) = P (exactly k contacts or exposures in (0, t)) $= F_k(t) - F_{k+1}(t)$ with $F_0(t) = 1$

$P(T > t) = \sum_{n=k}^{\infty} V_k(t) P(\sum_{i=1}^{n} x_i < y)$

$= \sum_{n=k}^{\infty}[F_k(t) - F_{k+1}(t)]\ [\, g*(\lambda^2)\,]^k$

$= [F_0(t) - F_1(t)] + [F_1(t) - F_2(t)]g*(\lambda^2) + [F_2(t) - F_3(t)]g*(\lambda^2)$

$= (1 - F_1(t))[1 - g*(\lambda^2)] - g*(\lambda^2)F_2(t)\,[1 - g*(\lambda^2)] + \cdots$

$= 1 - [1 - g*(\lambda^2)]\sum_{i=1}^{\infty} F_k(t)\ \ [g*$ *λ2k– 1.*

$L(t) = 1 - S(t) = P[T > t]$

$= [1 - g*(\lambda^2)]\sum_{k=1}^{\infty} F_k(t)\ \ [g*$ *λ2k– 1.*

Taking Laplace transformation of L (t) we ge

$$L^*(s) = \frac{1}{s}\, l^*(s)$$
$$= [1 - g * (\lambda^2)] \sum_{K=1}^{\infty} \frac{1}{s} \left(f^*(s)\right)^k\, g * (\lambda^2)^{k-1}$$

$$L^*(s) = [1 - g * (\lambda^2)]\, f^*(s) \sum_{k=1}^{\infty} [f^*(s)\, g * (\lambda^2)]^{k-1}$$

$$L^*(s) = \frac{[1-g*(\lambda^2)]\, f^*(s)}{[1-g*(\lambda^2)\, f^*(s)]}$$

$put \quad f^*(s) = \left(\frac{c}{c+s}\right)$ since $f(.) \sim \exp(c)$

$$l^*(s) = \frac{[1-g*(\lambda^2)]\left[\frac{c}{c+s}\right]}{\left[1-g*(\lambda^2)\left(\frac{c}{c+s}\right)\right]} = \frac{[1-g*(\lambda^2)]\left[\frac{c}{c+s}\right]}{\left[\frac{c}{c+s}\right]\left[\left(\frac{c+s}{c}\right)-g*(\lambda^2)\right]}$$

$$l^*(s) = [1 - g * (\lambda^2)] \left[\left(\frac{c+s}{c}\right) - g * (\lambda^2)\right]^{-1}$$

$$\frac{d}{ds} l^*(s) = \frac{d}{ds}[1 - g * (\lambda^2)] \left[\left(\frac{c+s}{c}\right) - g * (\lambda^2)\right]^{-1}$$

$$= [1 - g * (\lambda^2)] \frac{d}{ds} \left[\left(\frac{c+s}{c}\right) - g * (\lambda^2)\right]^{-1}$$

$$= [1 - g * (\lambda^2)](-1) \left[\left(\frac{c+s}{c}\right) - g * (\lambda^2)\right]^{-2} . 1$$

$$= [1 - g * (\lambda^2)] \left[\left(\frac{c+s}{c}\right) - g * (\lambda^2)\right]^{-2}$$

$$E(T) = \frac{-d}{ds} l^*(s) \;|_{s=0}$$

$$= -[1 - g * (\lambda^2)][1 - g * (\lambda^2)]^{-2} \times \frac{1}{c}$$

$$= \frac{[1-g*(\lambda^2)]^{-1}}{c}$$

$$E(T) = \frac{1}{c[1-g*(\lambda^2)]}$$

If $g(.) \sim \exp(\mu)$, $g * (\lambda^2) = \frac{\mu}{\mu+\lambda^2}$

On simplification we get

$$E(T) = \frac{1}{c\left[1-\frac{\mu}{\mu+\lambda^2}\right]} = \frac{\left[1-\frac{\mu}{\mu+\lambda^2}\right]^{-1}}{c}$$

$$= \frac{\left[\frac{\mu+\lambda^2-\mu}{\mu+\lambda^2}\right]^{-1}}{c} = \frac{\left[\frac{\lambda^2}{\mu+\lambda^2}\right]^{-1}}{c}$$

$$E(T) = \frac{\mu+\lambda^2}{c\,\lambda^2}$$

Now

$$E(T^2) = \frac{-d^2}{ds^2} l^*(s) = \frac{d}{ds}\left[-\left(1 - g * \lambda 2c + sc - g * \lambda 2 - 2\right)\right.$$

$$= -\left(1 - g * (\lambda^2)\right) \frac{d}{ds} \left[\left(\frac{c+s}{c}\right) - g * (\lambda^2)\right]^{-2}$$

$$E(T^2) = \frac{-d^2}{ds^2} l^*(s) \;|_{s=0} = 2\left(1 - g * \lambda 21 - g * \lambda 2 - 3 \times 1c\right.$$

$$E(T^2) = \frac{2\left(1-g*(\lambda^2)\right)^{-2}}{c^2}$$

$$E(T^2) = \frac{2}{c^2\left(1 - g * (\lambda^2)\right)^2}$$

$$= \frac{2}{c^2}\left[\frac{1}{\left(1 - \frac{\mu}{\mu + \lambda^2}\right)^2}\right] = \frac{2}{c^2}\left[\frac{1}{\left(\frac{\mu + \lambda^2 - \mu}{\mu + \lambda^2}\right)^2}\right]$$

$$E(T^2) = \frac{2}{c^2}\left[\frac{1}{\left(\frac{\lambda^2}{\mu + \lambda^2}\right)^2}\right] = \frac{2}{c^2}\left[\frac{\mu + \lambda^2}{\lambda^2}\right]^2$$

$$V(T) = E(T^2) - [E(T)]^2$$

$$= \frac{2}{c^2}\left[\frac{\mu+\lambda^2}{\lambda^2}\right]^2 - \left[\frac{\mu+\lambda^2}{c\,\lambda^2}\right]^2$$

$$= \frac{2}{c^2}\left[\frac{\mu+\lambda^2}{\lambda^2}\right]^2 - \frac{1}{c^2}\left[\frac{\mu+\lambda^2}{\lambda^2}\right]^2$$

$$= \frac{1}{c^2}\left[\frac{\mu+\lambda^2}{\lambda^2}\right]^2$$

$$V(T) = \frac{(\mu+\lambda^2)^2}{c^2\lambda^4}$$

5. Numerical illustration and conclusions:

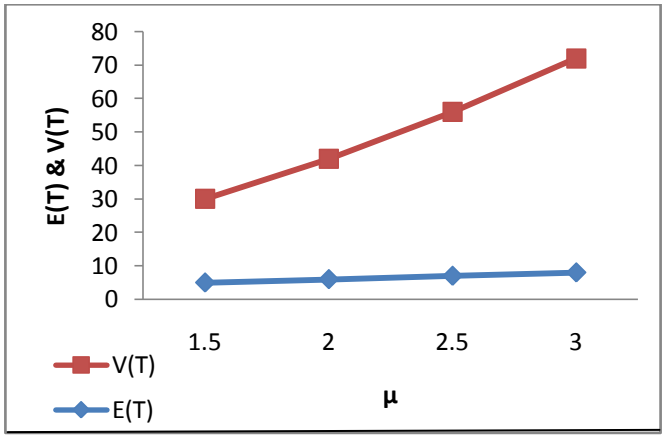

Figure - 1

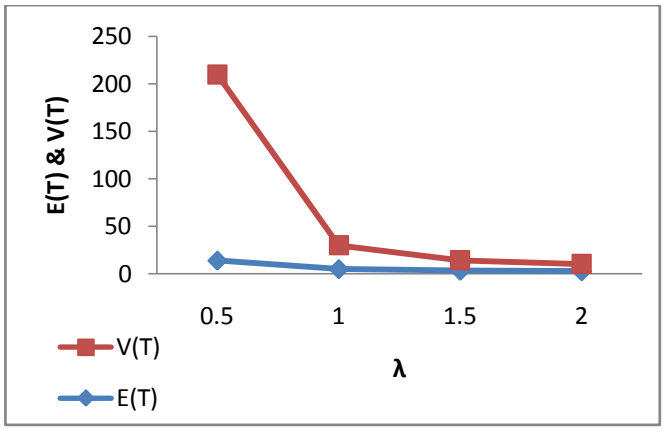

Figure - 2

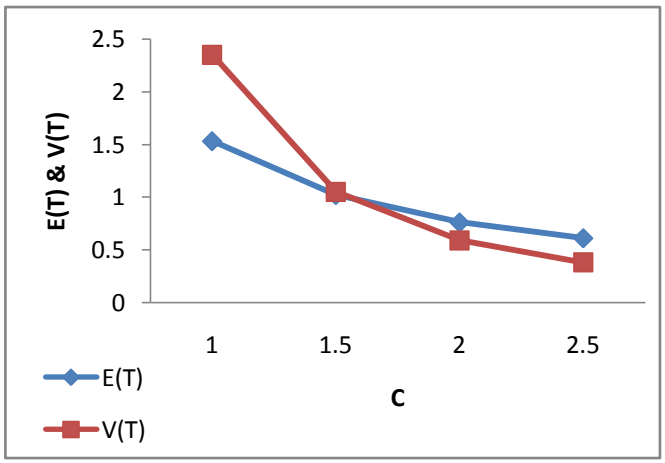

Figure - 3

On the basis of the numerical illustration the following conclusions can be drawn.

a) If the value of μ which is the parameter of the distribution of the random variable X which denotes in the contribution to antigenic diversity increases E(T) and V(T) both increase. This is due to fact that the random variable X follows exponential distribution with μ as parameter and therefore E(X) = $\frac{1}{\mu}$. As μ increases the average contribution to antigenic diversity decreases. Hence, E (T) shows an increase and V (T) also increases.

b) If c which is the parameter of the distribution of inter arrival times increases E(T) decreases and V(T) also. This is due to fact that the random variable *U* denoting the inter contact times follows exponential with parameter c. So E(U) = $\frac{1}{c}$. Therefore if c increases, E(U) decreases. Hence the time interval between successive contacts decline, therefore it takes less of times fare seroconversion.

c) λ which is the parameter of the distribution of the threshold which follows Two Parameter Burr Type X Distribution shows an increase then E(T) decreases but V(T) is an the increases.

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